臺灣東亞文明研究學刊　第18卷第1期（總第35期）　2021年6月　頁177-220

【研究論著】General Article　DOI:10.6163/TJEAS.202106_18(1).0005

Comparison of histories, history of comparison: A plea to re-investigate mathematical cases from India and China

歷史比較，比較歷史：試論重新檢視印度與中國數學案例的必要

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Keywords: indeterminate analysis, China, India, comparison, epistemic culture, cognitive studies

關鍵詞：不定分析，中國，印度，比較，認識論文化，認知研究

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Abstract

The comparative method is intrinsic to the history of science. Despite this fact, studies comparing Chinese and Indian mathematical texts remain few. Scholars have long noticed similarities between Chinese and Indian algebraic results and procedures, numeration systems and astronomy. Yet, these comparisons raise interesting questions for historiography: as mathematical texts are written in classical Chinese and Sanskrit, corpuses became representative of so-called 'Chinese mathematics' or 'Indian mathematics', thus reducing the concept of culture to nation or civilization.

Since Wylie's first comparative study in 1852, many scholars have focused on the resemblance between Chinese and Indian indeterminate equations. The analysis of India's contribution to the solution of indeterminate equations (kuṭṭaka) and the dayan 大衍 method of Qin Jiushao 秦九韶 constitutes an essential part of the historiography of the comparative study of mathematics in India and China. The aim of this article is twofold: 1) to investigate the construction of this history, in particular how the concept of transmission depends on prejudice regarding algorithms; and 2) to propose an alternative ways of comparison and show their promise. To reveal the heuristic dimensions of contrast, I am going incorporate recent studies on epistemic cultures as well as an example based on two medieval treatizes by Li Ye 李冶 and Nārāyaṇa and their relation to cognitive tasks.

摘要

比較研究法是科學史不可或缺的一部分，但以中國和印度數學文本進行的比較研究仍然不多。學者們長期以來都注意到中國和印度的代數成果、程序、計數系統和天文學間有著相似之處。然而，這些比較研究為史學提出了一個有趣的疑問：因為兩國的數學文本分別以文言文和梵文書寫，兩者的文本成為了所謂「中國數學」或「印度數學」的代表，從而將文化概念降為國族或文明概念。

自 Wylie 在一八五二年首次提出比較研究，許多學者不斷致力於研究中國和印度在不定方程組間的相似性。對印度在不定方程組（kuṭṭaka）的貢獻和解法，以及秦九韶大衍術的研究，都構成了印度和中國數學比較研究史的重要部分。本文具有雙重目的：1) 研究歷史書寫的構造，尤其概念傳輸是如何取決於算法的偏見；2）提出一個可替代的比較研究方法和展示其前景。研究者將結合近期有關認識論文化的研究、基於李冶和那拉衍那（Nārāyaṇa）中世紀文本例子和他們與認知研究的關係，以展示這項比較研究的啟發性面向。

I. Introduction

Comparing Chinese and Indian mathematical texts may sound like a strange enterprise. Scholars have long noticed similarities between Chinese and Indian algebraic results and procedures, and these similarities are an interesting question for historiography. The principal objective of a comparison is to determine the differences or convergences of two thoughts belonging to distinct linguistic, chronological, geographical, or cultural horizons. Comparisons mainly aim to identify possible influences, ramifications, contacts and mutual representations; sometimes, they are a tool of Universalist philosophy. Comparisons have heuristic and historical potential when used to identify similarities. By interrogating differences, comparisons contribute to questioning of certain 'obvious' concepts. They unveil an underlying structure of thought that is not self-evident. Here the purpose is strictly heuristic. I do not intend to add another comparative study. I wish to investigate the comparative method's potential dimensions and investigate how mathematics from China and India have historically been compared. Comparative methods can unveil different strategic objectives, like modes of systematization and styles of reasoning. A comparison of differences leads to a reflection on cultural aspects that are involved in mathematical activity. Furthermore, it encourages questioning the conceptual definition of "culture" and "tradition" when one faces mathematics written in different languages. These concepts are often still applied like labels applicable to the whole of China and India, as if there were only one culture or one continuous tradition to the present in China or India. The aim of this study is to challenge the view of mathematics as uniform fields relating to specific "cultures" by identifying the mosaic of mathematical practices or cultures of computation.

Connections between China and India focused on several aspects: Buddhism, mainly with figures like Yi Xing 一行 (8th century) and Fa Xian 法顯 (4th-5th

centuries), numeration systems, solutions to indeterminate equations, magic squares, arithmetical triangles and the broken bamboo problem. These aspects were always studied separately, and most of the time comparisons are used to claim the authorship of a procedure and the recognition of influences. When comparing the Indian and Chinese procedures, the most addressed topic the is the solution to indeterminate equations. The first mention of rules was stated in China during the 3rd century, followed with new development in India and ended with the Chinese remainder theorem by Qin Jiushao 秦九韶 during the Song dynasty (960 – 1279). After the re-edition of Qin Jiushao's works in the private library collection *Yijia tang congshu* 宜稼堂叢書 (Yi Jia Tang Collection) in 1842, a debate started concerning the connection between these procedures.

In this article, I will use the example of indeterminate equation to study the historiography of comparison whose purpose is to identify influences. Then I will present another promising hypotheses to show the potential of contrast, if understood as comparison of ramification or a heuristic comparison.

II. 1. Comparison of influence: Chinese and Indian treatises on indeterminate analysis

Similarities between procedures called Indeterminate Analyses have been a topic of research since the 19th century. The Chinese remainder theorem is a theorem of number theory, which states that if one knows the remainders of the Euclidean division of an integer *n* by several integers, then one can uniquely determine the remainder of the division of *n* by the product of these integers, under the condition that the divisors are coprime pairwise. The earliest known statement of the theorem, as a problem with specific numbers, appeared in the treatise *Sunzi Suanjing* 孫子算經 (*The Mathematical Classic of Master Sun*) compiled between the 3rd and 5th century:

> "Now, there is an unknown number of things. If we count by threes, there is a remainder of two; if we count by fives, there is a remainder of three; if we count by sevens, there is a remainder of two. Find the number of things."[1]

Sunzi's work does not, however, contain the full algorithm. The result was later generalized with a complete solution generally called *dayanshu* 大衍術 or more precisely *dayan zongshu shu* 大衍總數術 (Great Inference/Extension for All Numbers) and its core sub-procedure, *dayan qiuyi shu* 大衍求一術 (Great Inference/Extension to Find One) in Qin Jiushao's 1247 *Shushu Jiuzhang* 數書九章 (*Mathematical Treatise in Nine Sections*). Indeterminate analysis arose in China primarily as a method to calculate calendars.[2] Chinese astronomers had to solve systems of relationships with data so vast that it was impossible to get unique solutions without some special algorithms. The Chinese remainder theorem was one such algorithm. It is not clear when the Chinese began to investigate this type of calculation. Unfortunately, the complete calculation method used by astronomers contemporary to Sunzi has not been passed down to the present. Qin Jiushao gave the systematic description as a finished product in the 13th century.

Similar problems of calendar construction arose in India and the Islamic world due to calendar making and astronomical calculation issues. An algorithm for solving indeterminate equation was described by Āryabhaṭa (5th or 6th century). It is in Āryabhaṭa's work, *Āryabhaṭīya, Ārya*-*siddhanta*, that we come across the first unequivocal discussion of the subject of indeterminate analysis. It arose, just as the previous theorem did in China, in the field of astronomy, where there is a

1 Lam Lay Yong and Ang Tian Se translation. From Sunzi suanjing Ch.3, prob.26. 今有物，不知其數。三、三數之，賸二；五、五數之，賸三；七、七數之，賸二。問物幾何？Lam Lay Yong and Ang Tian Se, *Fleeting Footsteps: Tracing the Conception of Arithmetic and Algebra in Ancient China*,Revised edition (Singapore :World Scientific,2004),p. 139.

2 Li Yan and Du Shiran ,*Chinese Mathematics: A Concise History*, Translated by Crossley J.N and Lun AA. W-C., (Oxford: Clarendon Press,1987), p.94.

need to determine the orbits of planets.[3] Special cases of the Chinese remainder theorem were also known to Brahmagupta (7th century) *Brāhmasphuṭasiddhānta*, Bhāskara I (7th century) *Āryabhaṭīyabhāṣya,* Mahāvīra (9th century) *Gaṇitasārasaṅgraha*, Bhāskara II (12th century) *Siddhānta Shiromani, Lilāvatī* and later appear in Europe in Fibonacci's *Liber Abaci*.[4]

In India, the method used to solve an indeterminate equation is called *kuṭṭaka*. It is a method for solving equations in the form of $ax + c = by$, with whole numbers. The method is based on the division of a by b, b by the remainder of the previous division, etc. Numbers are decreasing, and after several operations, the remainder is zero. As the number of solutions is infinite, the purpose is to find the smallest positive solutions for x and y.

The Sanskrit words *kuṭṭa, kuṭṭaka, kuṭṭākāra* are derived from the root, meaning *kuṭṭ*, "to crush", "to grind", "to pulverise". Gaṇeṣa in a commentary to Bhāskara's *Lilāvatī* explains:

> "*kuṭṭaka* is a term for the multiplier, for multiplication is admittedly called by words importing 'injuring', 'killing'. A certain given number being multiplied by another [unknown quantity], added by a given divisor leaves no remainder; that multiplier is the *kuṭṭaka*: so it has been said by the ancients. This is a special technical term." [5]

It has been observed that the subject of indeterminate analysis of the first degree was considered so important in India that the whole science of algebra was once named after it. Āryabhaṭa II enumerates it distinctively along with the

3 Joseph,G.G.,*The Crest of the Peacock. Non-European Roots of Mathematics*, 3rd edition (Princeton and Oxford:Princeton University Press, 2011), p.386.

4 Leonardo Fibonacci, *Liber Abaci* (1202), Laurence E.Sigler (trad.), *Fibonacci's Liber Abaci : A Translation Into Modern English of Leonardo Pisano's Book of Calculation* (Springer-Verlag, 2002).

5 Datta, B. and Singh, A. N.,*History of Hindu Mathematics, A Source Book*, Part II, (Bombay, India: Asia Publishing House,1935), p.90.

sciences of arithmetic, algebra and astronomy. So did Bhāskara II and others. The treatment of indeterminate analysis is sometimes presented as a last section of *pātīgaṇita* (literally "board-computation," sometimes translated as "arithmetic"). However, it belongs to *bījagaṇita* (literally "seed-computation," sometimes translated as "algebra"), like in the case in *Bījagaṇitāvataṃsa* by Nārāyaṇa (14th century) or *Bījagaṇita* by Bhāskara II. In contrary to the scarcity of Chinese evidence, there is abundant literature in India. For solving the same type of problems, China provides the earliest evidence, while India furnishes an abundance of examples, thus drawing attention to the question of similarity, difference and influence of the two procedures.

In Europe, the notion of congruences was first introduced and used by Carl Friedrich Gauss (1777-1855) in his *Disquisitiones Arithmeticae* of 1801. Gauss illustrates the Chinese remainder theorem on a problem involving calendars, namely, "to find the years that have a certain period number with respect to the solar and lunar cycle and the Roman indiction." [6] Gauss introduced a procedure for solving the problem that had already been used by Leonhard Euler (1707-1783) but was in fact an ancient method that had appeared several times: the problem of calendar making attracted the attention of astronomers from Sunzi to Qin Jiushao (this period includes the Indian development) whose statement of the rule for the general solution of the indeterminate equation of the first degree predates the work of Euler and Gauss by at least five hundred years.

1. The historiography

Since the 19th century, many scholars have focused on the resemblance between the Chinese remainder theorem and the Indian solution to indeterminate equations. The study of India's contribution to the solution of indeterminate

6 Indiction is a fiscal period of fifteen years used as a means of dating events and transactions in the Roman Empire, in the papal and some royal courts, used from the 4th century onwards until as late as the 16th century.

equations (*kuṭṭaka*) and the *dayan* 大衍 method of Qin Jiushao represent an important part of the historiography of the comparative study of mathematics in India and China.

This happened in the specific context of the rediscovery of early Chinese mathematics in China: starting in the 18th century Chinese scholars gathered ancient and medieval texts intending to recover Chinese knowledge in reaction to Western knowledge brought by the Jesuits. In 1838 and 1839, some samples of mathematical knowledge from China arrived in Europe thanks to the works of G. Libri and E. Biot. From August 1852, the British Protestant missionary and sinologist, Alexander Wylie (1815–1887) published nine instalments of an account entitled *Jottings on the Science of the Chinese Arithmetic* in the newspaper *North China Herald*. Wylie could provide a more documented work because he had access to materials in China and was in contact with the most important mathematician of that time, Li Shanlan. Wylie's series of article played a pioneering role in studying of the history of Chinese mathematics in the Western world. It is now considered the only reliable source on the history of Chinese mathematics that preceded the publication of Yoshio Mikami's *The Development of Mathematics in China and Japan* in 1913.[7] According to Wylie, the series' objective was to clarify some erroneous statements about the status of mathematics in China found in the Western publications of that time. Westerners in the 19th century thought that the Chinese possessed only limited mathematical knowledge. In order to respond to the claim that there was no algebra in China, Wylie provided some algebraic methods, the most well know one being the *dayan*. This series was translated into German by Biernatski in 1856 and later into French by Terquem in 1863.[8] These translations were more accessible than the original source published in Shanghai. They thus became influential for historians at the end of the 19th

7 Dauben, J. W. (ed.),*The History of Mathematics from Antiquity to the Present: A Selective Annotated Bibliography* , (New York: Garland,1985); Siu Man-Keug & Chan Yip-cheung, "On Alexander Wylie's Jotting on the science of the Chinese Arithmetic," *Hisrory and Pedagogy of Mathematics 2012*, (Daejeon, Korea: DCC, July 16-20 2012).

8 Terquem, O., *Nouvelles Annales de Mathematiques* (2e ser., 1863), p.35.

century and in the early 20th century like Hankel, Zeuthen, Vacca and Cantor. The translations were not free from mistakes and the historians, who had no access to Chinese sources, attributed the mistakes to the Chinese authors instead of to the translators.[9]

Some scholars were certain that the *dayan* method was derived from the *kuṭṭaka* since Wylie's first study based on an article written in 1817.[10] The majority of these studies are dedicated to paternity questions or to universalist proofs. That is, either one wanted to show that a country had first made a discovery and influenced other countries, or one wanted to show the universality of algebra by showing its existence in every country despite different methods of expression. Comparisons of points of resemblance were used to focus on influences and circulation of knowledge. In 1973, Ulrich Libbrecht[11] following Yushkevich,[12]concluded that it makes no sense to accept the idea of a historical relationship between the Chinese *dayan* procedure and the Indian *kuṭṭaka*, and that the resemblance was superficial, which is also the conclusion of G.G. Joseph.

"There are four currently accepted approaches to solving the astronomy problem: (1) An arithmetic approach whose result is laborious and restricted; (2) The approach that originated with Āryabhaṭa, refined by Brahmagupta, Mahāvīra, and Bhāskara II, referred to as *kuṭṭaka* and which consists of continuous divisions and substitutions; (3) The method favoured in recent time, with is close to the Indian one; and (4) The Chinese procedure (*dayan*)."[13]

9 Martzloff, J. C., *Histoire des Mathématiques Chinoises* (Paris : Masson,1988).

10 Wylie, A ,"Jotting on the Science of the Chinese: Arithmetic," *North China Herald*, (Aug.-Nov, 1852), p.185.

11 Libbrecht,U., *Chinese Mathematics in the Thirteenth Century: The Shu-shu chiu-chang of Ch'in Chiu-shao* (Cambridge, Massachusetts :The MIT Press,1973),pp. 220-222, 359-366.

12 Yushkevich, A. P., "O dostizenijax kitajskix ucenyx v oblasti mathematiki," (On the achievements of Chinese scholars in the field of mathematics), in *Iz Istorii Nauki I Texniki Kitaja* (Essays in the History of Science and Technology in China), Moscow, p.130.

13 Joseph,G.G.,*The Crest of the Peacock. Non-European Roots of Mathematics*, p.283-285.

It would seem that this conclusion is the prevalent one nowadays.

In Europe, this history started with a publication in *Journal Asiatique* by Biot[14] who first mentioned the Chinese remainder theorem. However, it seems that the publication remained unnoticed. Alexander Wylie's 1852 "Jotting on Science of the Chinese" in the *North China Herald*, on the other hand, gathered considerable momentum. It was the first time that Qin Jiushao's rule by was explained in detail. Wylie's publication includes a full explanation of Qin Jiushao's first problem and some notes on other problems. Ten years earlier, Qin Jiushao's works were republished in the *Yijia tang congshu* 宜稼堂叢書 in China, and this may have enabled Wylie to study the *dayan* procedure.

Wylie[15] was the first to state that "[t]his appears to be the formula, or something very like it, which was known to the Hindoos under the name of Cuttaca, or as it is translated 'Pulverizer,' implying unlimited multiplication, which is not far from the meaning of the ta-yen or 'Great Extension'". Libbrecht[16] identified his source as the *Edinburgh Review* and claims that Wylie's source is insufficient for proving a similarity between the two procedures.

Cantor already acknowledged this insufficiency[17]: "It is also a fact that this (*dayan*) method is absolutely different from the Indian pulverization, with which [scholars] liked to compare it before they understood it".[18] In addition, Biernatzki[19] translated Wylie statement into German and added : "However it does not follow that the Chinese received their arithmetical researches ready-

14 Biot, E.,"Table générale d'un ouvrage chinois intitule Souan-Fa Tong-Tsong, ou traité complet de l'art de compter," *Journal asiatique*, 3rd series, No.7, (1839), p. 207.

15 Wylie, A ,"Jotting on the Science of the Chinese: Arithmetic," p.185.

16 Libbrecht,U., *Chinese Mathematics in the Thirteenth Century: The Shu-shu chiu-chang of Ch'in Chiu-shao*, p.359.

17 Cantor, M., *Vorlesungen über die Geschichte der Mathematik*, vol. 1, 1880-1908, p. 587.

18 Es steht eben so fest, dass dieses Verfahren von der indischen Zerstäubung, mit welchem man es zu vergleichen liebte, bevor man es verstand, durchaus verschieden ist…

19 Biernatzki, K.L. ,"Die Arithmetik der Chinesen,"*Journal für reine und angewandte Mathematik*, 52 (1856) , p.83.

made from the Indians, or borrowed from them their elements of arithmetic, especially the Great Extension rule".[20]Despite common points, both Cantor and Biernatzki argue that the two procedures are not related.

It is in 1874 that Hankel[21] reanimated the debate: "If any proof is still necessary that a very close interconnection exists between Indian and Chinese mathematics, then this can be found in the ta-yen rule (*dayan*), which is mentioned in the third century A.D. in the Suan-ching of Sun Tzu (*Sunzi suanjing*) and which was discussed in detail at the beginning of the eight centuries in the Ta-yen li-shu (*dayan lishu*), the work of an Indian Buddhist monk … This ta-yen (*dayan*) is ultimately nothing other than the Indian *kuṭṭaka,* which was precisely the same applications to chronology and calculations of conjunctions".[22] Libbrecht notices the absence of argument for all of these statements and the confusion between the Buddhist monk Yixing and the *Yi-jing* 易經 (Book of Change).

The first argumentation based on a full explanation appeared in Matthiessen[23] who demonstrated that the *dayan* and *kuṭṭaka* methods are two different methods. According to Matthiessen, the Indian *kuṭṭaka* agrees with the method of Bachet de Mérizac, whereas the *dayan* is the same method as Gauss's congruences. Libbrecht concluded that "Only an internal analysis of both methods is able to yield a scientific treatment of the question". However, Matthiessen did not access any

20 Daraus folgt aber nicht, dass die Chinesen ihre arithmetischen Forschungen von dem Hindus fertig übernommen, oder von ihnen Elemente dere Arithmetik, insbesondere die grosse Erweiterungsregel, entlehnt haben.

21 Hankel, H., *Geschichte der Mathematik im Altertum und Mittelalter*(Leipzig,1874), p.407.

22 Wenn es noch eines Beweises bedürfte, dass zwischen indisher und chinesischer Mathematik der engste Zusammenhang besteht, so ist die Regel Ta jan (= Grosse Erweiterung), die schon im 3. Jahrhundert n. Chr. In den Suan-king (= arithemtisher Klassiker) des Sun-Tse vorkommt, und im Anfange des 8. Jahrhunderts ausführlich behandelt wurde in dem Ta jan li shu (=Sehr erweitertes Himmelzeichenbuch), dem Werke eines indisher [!] Buddhapriesters… Diese Regel Ta jan ist aber nichts anderes, als die indischen *kuṭṭaka,* von der hier ganz dieselben Anwendungen auf die Chronologie und die Berechnung gewisser Constellation gemacht warden wie dort…".

23 Matthiessen, L. ,"Vergleichung der indischen Cuttaca und der chinesischen Tayen-Regel," in *Sitzungsberichte der mathematisch-naturwissenschaftlichen Section in der 30. Versammlung deutscher Philologen und Schulmänner in Rostock*, *1975, 75* (Leipzig, 1876).

documents other than Biernatzki's translation. In fact, he could not know the Chinese method of solving the congruence as he had no access to the Chinese texts.

The opposition between Hankel and Matthiessen was well known to the Japanese scholar Hayashi who in 1905 wrote: "Hankel … shows us that the da-yan rule for the solution of indeterminate equations is the same as the cuttaca d'hyana of Indian mathematics, while Matthiessen … shows us that the cuttaca is the method of continued fractions and the da-yan rule is the method of congruences of Gauss".[24] However, lots of literature derives from another source from Europe.

Van Hee was the second European sinologist after Wylie to have access to Chinese sources. Between 1911 and 1913, he published a series of articles on Chinese mathematics, including a special article on indeterminate analysis: "The foreign influence manifests itself: (a) the numbers are written from left to right, horizontally; (b) the ta-yen (*dayan*), or formula for solving indeterminate problems, resemble the Indian *kuṭṭîkara"*. Libbrecht[25] provides valuable details to show how Van Hee's contributions are highly debatable. On the basis of a few problems riddled with many misunderstandings, he derived Qin Jiushao's work from India. His influence on other historians of mathematics was important.[26] For instance, as Needham[27] noticed, G. Loria's articles published between 1921 and 1929 are a

24 Hayashi, T. ,"Brief History of Japanese Mathematics," *Nieuw Archief voor Wiskunde*. 6(1905),p.310.

25 Libbrecht,U., *Chinese Mathematics in the Thirteenth Century: The Shu-shu chiu-chang of Ch'in Chiu-shao*, pp.319-322.

26 Libbrecht showed how the influence was not constructive. Van Hee spreads the idea that there is no generality and synthesis in Chinese text : "Chez les mathematiciens jaunes, c'est l'amour du detail, sans grand soucis de la synthese" ("in the mind of yellow mathematician, there is love of detail, without great concern for synthesis").Van Hee.,"L'algèbre chinoise," *Toug Pao*, 13(1912), p.291. "J'ajoute que si les livres des mathematiques chinois de toute epoque disparaissaient, la science n'y perdrait rien comme mathematiques" (« I say in addition that if the works of the Chinese mathematicians of every era were to disappear, science would lose nothing in the way of mathematics »). " Le classique de l'ile maritime, ouvrage chinois du IIIe siècle," *Quellen und Studien zur Geschichte der Mathematik* (Part B: Astronomie und Physik), 2(1932), p.259.

27 Needham, J.,*Science and Civilisation in China* (Cambridge,1954), vol.3, p.1, note e.

restatement of Van Hee's publication.[28] Loria stated that Qin Jiushao produced "a treatise in which one recognizes foreign influence".[29] As Needham says: "he suffered from an invincible suspicion that the Chinese must have borrowed all their ancient techniques from the West".[30] Libbrecht reaches the same conclusion and mentions another article published in 1931 by Smith who refers to Van Hee, using the same examples found in Loria and doubting the Chinese procedure authenticity.[31] Smith's mistakes are also repeated in Becker and Hofmann's *Geschichte der Mathematik*.[32]

There was no need to wait until 1951 for the first critique written in a Western language to the conjectural attribution of Indian origin to Chinese mathematics. The Japanese historian Mikami said in 1913[33]: "the possibility of the Chinese mathematics having influenced by the science in India may well be conjectured from the meager account here given [that is about I-hsing (*Yixing*)]. As for exact information, we have none". Afterwards there were no reactions from Loria towards Mikami's critic. Cajori's article on Chinese mathematics[34]was based on Mikami's findings. Historians who were not affected by Van Hee and Loria's conclusions disconnected the Chinese remainder theorem from the Indian

28 Loria, G., "Documenti relativi all'antica matematica dei cinesi,"*Archeion*, 3(1922), pp.141-149; Loria, G., *Storia delle matematiche dall' alba della civiltà al secolo XIX*, 3 vols (Turin, 1929).

29 Loria, G., "Documenti relativi all'antica matematica dei cinesi,"*Archeion*, 3(1922), pp.141-149.

30 Needham, J.,*Science and Civilisation in China* (Cambridge,1954), vol.3, p.1, note e.

31 Libbrecht,U., *Chinese Mathematics in the Thirteenth Century: The Shu-shu chiu-chang of Ch'in Chiu-shao*,pp. 323-324; Smith, D.E.,"Unsettled Questions concerning the Mathematics of China," *The Scientific Monthly*., Vol. 33, No. 3 (Sep., 1931), pp. 244-250.

32 Becker and Hofmann, *Geschichte der Mathematik*,1951.

33 Mikami, Y., *The Development of Mathematics in China and Japan* (Abhandlungen zur Geschichte der mathematischen Wissenschaften, n.30), Leipzig, 1921 (reviewed by H. Bosmans, 1913).

34 Cajori, F., *A History of Mathematics*, 2nd edition (New York,1919),pp. 71-77.

procedure. Tropfke[35]repeats Matthiessen statement that *dayan* and *kuṭṭaka* are different, while Sarton[36]deals with Qin Jiushao without mentioning influence.

In the 1950's the debate on influences between India and China was far from over, but the argument in favor of influence and similarity became quite different. Yushkevitch, who had access to original sources and new Chinese scholar works like Li Yan and Qian Baocong, produced the most important modern study prior to Needham's *Science and Civilization in China*. He gave an objective description of Chinese mathematics without any preconception about the level of advancement they were supposed to reach or not. In 1964[37], he stated: "The Indian method is of course quite different from the Chinese one"[38] confirming Matthiessen's idea. This does not mean that the debate was over because on the other side, Needham[39] claimed that "the Chinese *dayan* procedure was similar to the *kuṭṭaka* ... method in Indian mathematics…" and "the argument of Matthiessen that they were very different does not carry conviction".

The debate about the possible influences among procedures naturally finds echoes in India, but the debate is not focusing on relation with China. Instead, it is the so-called "Western" influence that will make Indian historians of mathematics react. H. Kern (1833-1917) had edited Āryabhaṭa's *Āryabhaṭīya* in 1874.[40] The first translation in a European language was made Leon Rodet (1850-1895), in French, in 1879,[41] and the first English translation appeared much later, thanks to P. C. Sengupta (1876-1962) in 1927. However, Rodet's interpretation is now considered faulty. G. R. Kaye, member of the Department of Education of the

35 Tropfke, J.,*Geschichte der Elementarmathematik*,7 vols (Leipzig,1921-1924).

36 Sarton, G., *Introduction to the History of Sciences*, 3 vols, No. 376. (Baltimore:Carnegie Institution Publ,1927-1947).

37 Yushkevitch, A.P., *Geschichte der Mathematik im Mittelalter* (tr. from Russian) (Leipzig, 1964), p.145.

38 Allerdings ist die indische Methode von der chinesischen verschieden"

39 Needham, J.,*Science and Civilisation in China* (Cambridge,1954), vol.3, p.122, note c.

40 Kern, H., *The Aryabhaṭīya with the commentary of Bhaṭadīpika of Parameśvara* (Leiden,1874).

41 Rodet, L., *Leçons de calcul de Aryabhata*(Paris: Imprimerie nationale,1879).

Government of India (Simla in North India) wrote an interpretation in 1908 of the Indian method.[42] Passionate about history of astronomy, mathematics and astrology, he produced many influential publications. His purpose was to show that "the work of Indian mathematicians from Āryabhaṭa to Bhāskara are essentially based on Western knowledge".[43] By "Western knowledge" he means the Greco-Latin origin transmitted through Arabic intermediaries. D. E. Smith, Cajori and Sarton all spread Kaye's interpretation by publishing his findings. Similarly, Majumdar[44] adopted Kaye's faulty reading. Majumdar argued that sophistication can be a criterion: « I absolutely fail to see how the Chinese method can stand in comparison with, or can be taken as the basis of, the elaborate process of the Indians».[45] 1927 seems to be a turning point in India: Indian scholars took the stage to argue with Kaye. Sengupta published the English translation of *Āryabhaṭīya* and proposed an interpretation of solution to indeterminate analysis based on Brahmagupta.[46] In 1930, Clark[47] relied on Parameśvara while Datta[48] and Ganguli[49] referred to Bhāskara I. The comparison with China was not their main preoccupation. However, it was Ganguli who concluded in 1931 on the topic: "… that this method (*dayan*) is different from the Indian methods…".[50] Nonetheless in 1964, S.N. Sen stated that "if there was any borrowing between

42 Kaye, G. R., "Note on the Indian Mathematics. No. 2. Aryabhaṭa," *Journal of the Asiatic Society of Bengal* IV 8(1908), pp. 111-114.

43 Kaye, G. R., *Indian Mathematics*(Calcutta, 1915).

44 Majumdar, N. K. ,"Aryabhata's rule in relation to indeterminate equations of the first degree," *Bulletin of the Calcutta Mathematical Society*,3(1911-1912).

45 Majumdar, N. K. , "On Chinese Indeterminate Analysis," *Bulletin of the Calcutta Mathematical Society*, 5(1913-1914), p.11.

46 Sengupta, P. C. ,"The Aryabhatiyan, translation, "*Journal of the Departments of Letters of the University of Calcutta*, XVI (1927).

47 Clark, W. E., *The Aryabhatiya of Aryabhata, translated into English with notes*(Chicago: University of Chicago Press,1930).

48 Datta, B., *The Sciences of the Sulbas: A study in early Hindu geometry* (Calcutta: Calcutta University Press,1932).

49 Ganguli, S.,"Notes on Indian Mathematics: A criticism of G. R. Kaye's interpretation," *Isis* 12(1929),pp.132-145.

50 Ganguli, S.,"India's contribution to the Theory of Indeterminate Equations of the Frist Degree," *The Journal of Indian Mathematical Society* (Madras),19(1931), pp.113.

China and India, it was not India but China at the receiving end”.[51] Sen’s chief aim is to prove the priority of Indian mathematics. “Moreover, SunTzu gave an example with answer; Āryabhaṭa I, on the other hand, or more correctly, the school to which he belonged, gave correct and general rules for the solution of both linear and simultaneous indeterminate equations. The chronological argument of about a hundred years between the time of Sun Tzu and Āryabhaṭa I, to which some emphasis has been given, is hardly of any significance in the view of the interest already referred to of the Vedic Hindus in indeterminate problems”.[52] The argument is based on the anteriority of the Vedic tradition.

The Chinese perspective is well summarized by Li Yan and Du Shiran[53]: “Indian astronomy passed into China, but its sorts of methods did not arouse the interest of Chinese astronomers and mathematicians […] The knowledge of mathematics and astronomy introduced from India did not have a great influence on Chinese mathematics and astronomy” [54]because the pen and paper computation were found too complicated compared to counting rods in the Tang dynasty. They admitted that there might have been exchanges “in both directions” during the Sui and Tang dynasties (581-907 AD), but none of these exchanges left substantial traces. Li Yan devoted several publications to Qin Jiushao and the *dayan* method, as well as Qian Baocong.[55] More recent publication by Lu Peng and Ji Zhigang[56]also states that the two procedures are independent. Their computation

51 Sen, S. N. ,“Study of Indeterminate Analysis in Ancient India”. *Proc. 10th International Congress of the History of Sciences* (Ithaca, 1962), p.493.

52 Yushkevitch, A.P., *Geschichte der Mathematik im Mittelalter* (tr. from Russian) (Leipzig, 1964), p.493.

53 Li Yan and Du Shiran ,*Chinese Mathematics: A Concise History*, p.168.

54 Li Yan and Du Shiran ,*Chinese Mathematics: A Concise History*, p.108.

55 Li Yan, ~~1958, 204~~ *Zhongguo kexue dagang* 中國數學大綱[Outlines of Chinese Mathematics],2 vols (Beijing: Kexue chubanshe,1958),p.204. Among other of his numerous collections. Qian Baocong, ~~1966. 60-103~~ *Song yuan shuxue lunwenji* 宋元數學史論文集[Collection of essays on the history of Song and Yuan mathematics] (Beijing,1966),pp.60-103.

56 Lu Peng 呂鵬 and Ji Zhigang 紀志剛, “Yindu kutaka xiangjie ji qi yu da yan zongshu shu bijiao xin tan” 印度庫塔卡詳解及其與大衍總數朮比較新探 [Detailed Explanation of Kutaka, India and a New Probe into Comparison with Dayan Summarization],《自然科學史研究》*Researches in the History of Natural Sciences*, Vol.38, Issuse 2(2019), pp. 172-188.

structures and history of changes in computation are too different. If there are similarities, it is a coincidence. There are no mentions of possible relations with India on the topic of solution to indeterminate equation. In general, only the discussion on the origin of the numeration system for great numbers in Buddhist texts find its ways into these publications.

In the short history presented above we see that what is labeled "Chinese" is a set of texts written in traditional Chinese script and what is labeled as "Indian" is - mainly written - Sanskrit culture. Other Indian languages and vectors of transmission that are not books were dismissed. There are only very few publications about Tibet, and scientific practices do not seem to be in the scope of history of mathematics. For instance, China is seen as a continuous whole as if nothing changed during centuries. What is called "traditional Chinese mathematics" cover different practices and methods. For instance, the method for setting up and solving cubic equations in the *Jigu suanjing* 缉古算经 (*The Continuation of Ancient Mathematics*) written in the time of Tang dynasty (618-907) is essentially different from the Procedure of Celestial Source used in Song dynasty (960-1279) .[57] As well, the interpretation of what is an equation in the Procedure of Celestial Source is not the same in Song and Qing dynasty (1644-1912).[58] Seeing the diversity of mathematics in Tang, Song and Qing dynasty as simply 'Chinese' is an oversimplification leading to historical nationalism.

Mathematics is essentially seen as a written product made to convey readymade procedures. The names 'China' and 'India' are applied like labels as if there were only one culture in China or India and as if historically the two geographical areas corresponded exactly to the modern notion we have of China

57 Lim, T. S. L. and Wagner D. B. ,"The Continuation of Ancient Mathematics: Wang Xiaotong's Jigu suanjing,"*Algebra and Geometry in the 7th-Century China* (Copenhagen: NIAS Press, 2017).

58 Pollet. C and Ying J-M.,"One quadratic equation, different understandings: the 13th century interpretation by Li Ye and later commentators in the 18th and 19th centuries" *Journal for History of Mathematics*,Vol. 30 No.3 (June 2017), pp. 137-162.

and India as nations. Furthermore, knowledge is reduced to particularism (ex. "Chinese traditional mathematics" or "Vedic mathematics"), essentialized and reduced into distinct disciplines when, in fact, both mathematical methods were part of astronomical practice. The more we read about mathematical comparison, the less we see astronomy. Mathematics is artificially separated from astronomy as if they the two had not once been connected. Also, author-to-author transmission seems to only happen through an exchange of written materials. Other practices are not taken into account and one should not forget that practices were not confined by the boundaries we anachronistically term "civilizations". Libbrecht describes beautifully how : "historical nationalism rises where historical science is unable to provide evidence, in default of historical data".[59] History of science practiced in a certain way contributes to shaping collectives that perceive themselves as communities.

2. The question of transmission: algorithm versus deduction

Even if Libbrecht is aware of nationalism hidden in comparative history of mathematics, he is not immune to other prejudices. Libbrecht[60] stated that the historical data are insufficient to lead to a scientific conclusion on a relation between the two methods. He relies on an "internal analysis"[61] to conclude that "it makes no sense to accept the idea of historical relationship between [the two methods]" and remains unconvinced of any potential relations. His idea of transmission is the following:

"It may be considered as a general rule that an *algorithm* borrowed from a *foreign country* is preserved more or less in its original state, and in many cases

59 Libbrecht,U., *Chinese Mathematics in the Thirteenth Century: The Shu-shu chiu-chang of Ch'in Chiu-shao*, p.362.

60 Libbrecht,U., *Chinese Mathematics in the Thirteenth Century: The Shu-shu chiu-chang of Ch'in Chiu-shao*, p.362.

61 Libbrecht,U., *Chinese Mathematics in the Thirteenth Century: The Shu-shu chiu-chang of Ch'in Chiu-shao*, p.362.

even the numbers are not changed. The reason seems to be that an algorithm, as it is not part of a *general deductive system*, is not derived easily from another algorithm. This means that, if an algorithmic rule is transmitted, there may be changes in the problems themselves, but seldom in the method; and when *two different cultures* make use of different patterns to solve the same problem, it is entirely wrong to deduce the first method from the second, even if this is possible in our deductive systems". [62]

This kind of argument raises several problems.

First of all, what exactly is an "internal analysis"? This phrase seems like a transcription into modern mathematical language. Indeed, it seems impossible not to translate an ancient mathematical text into modern terms. Everyone is aware now that modern mathematical description is a didactic tool to help to communicate mathematical content. Thus, modern concepts should be carefully handled as they cannot recover ancient concepts in their entirety. What may seem the same object within modern mathematical transcription is granted a different status in various sources. It is crucial that the history of mathematics not only describe the evolution of procedures, but also the evolution of the understanding of mathematical objects. For example, there have been different concepts of equations available in the world, and they have been identified as the same object only retrospectively because our analysis of ancient sources uses contemporary concepts that were designed through their synthesis. The use of modern mathematical terminology solely for the purpose of comparison leads to a standardization of practice under the criteria of our contemporary reading and prevents the valorization of the specificities of each text. Yet, it is also important to keep mathematical transcription in order to compare it with textual analysis. A text is not always a narration, a description or a presentation; it is a demonstration of results and concepts and thus contains traces of activities linked to their

62 Libbrecht,U., *Chinese Mathematics in the Thirteenth Century: The Shu-shu chiu-chang of Ch'in Chiu-shao*, p.362.

interpretation. Literal translation should complete modern mathematical transcription as it implies taking into account the way in which mathematical objects manifest themselves inside the text and the relations induced by the way of "talking about" these objects. "Internal analysis" is necessary but not sufficient.

More interestingly, Libbrecht's conclusion is based on the assumption that algorithm cannot be transmitted the same way as axiomatic-deductive reasoning. His idea is that an algorithm cannot be transformed or reinterpreted. The reasoning seems to be that an algorithm is not part of a general deductive system and is not derived easily from another algorithm. This means that if an algorithmic rule is transmitted, there may be changes in the problems themselves, but seldom in the method. According to Libbrecht, when two different "cultures" make use of different patterns to solve the same problem, it is wrong to deduce the first method from the second even if this is possible in deductive systems. The problem is that by opposing calculation and proof, one is reducing the scope of mathematical style of thinking to 'axiomatic-deductive' and to a certain type of proof. This conception of algorithm mistakenly reduces algorithms to method by repetition or recipe. In mathematics education it has become clear that changing data implies changes in method too, and vice et versa. These are practices of verification and demonstration inside algorithms. [63]Articulation of a list of operations also conveys mathematical meaning.[64] Libbrecht's argument results from the modern division between applied and theoretic mathematics, where the first is often undermined.

63 Pollet, C. ,"Reading Algorithms in Sanskrit: How to Relate Rule of Three, Choice of Unknown and Linear Equation?," in A. Volkov and V. Freiman (eds.), *Computations and Computing Devices in Mathematics Education before the Advent of Electronic Calculators* (Mathematics Education in the Digital Era Series, Springer Publishers,2018); Pollet, C.,"Interpreting Algorithms written in Chinese from the Point of View of Comparative History," in A. Volkov and V. Freiman (eds.), *Computations and Computing Devices in Mathematics Education before the Advent of Electronic Calculators* (Mathematics Education in the Digital Era Series, Springer Publishers,2018).

64 Chemla, K.,"What is the content of this book? A Plea for Developing History of Sciences and History of Text Conjointly," Chemla Karine (ed), *History of Science, History of Text*, (Dordrecht-Boston-London: Kluwer Academic Publishers,2004) pp. 201- 230; Pollet, C. , *The Empty and the Full: Li Ye and the way of mathematics* (Singapore:World Scientific,2020).

This modern division also applies to "cultures" assimilated to country and frontiers seen through a contemporary lens. India and China are still both seen uniform entities with deep borders. Furthermore, some scholars have drawn artificial boundaries between proof, deduction and algorithm. In order to redress this situation, one needs to reinvestigate the division between cultures, calculation and proof.

II. Comparison of ramification: From the point of view of epistemic culture

Recently, Zhu Yiwen[65] has demonstrated that Qin Jiushao was also versed in astronomy and how his calendrical knowledge was linked to other knowledge (e.g. accounting and divination). Zhu Yiwen talks about "culture of computations", borrowing the concept from Chemla and Fox-Keller, [66]to show that Qin Jiushao was confronted with two types of practices of different cultures and that his writing reflects how he dealt with this.

Zhu Yiwen showed that there were several *dayan* methods. One method is named Great Inference Procedure to Find One (大衍求一術 *dayan qiuyi shu*) and another is named Great Inference Procedure for All numbers (大衍總數術 *dayan zongshu shu*). The Procedure to Find One is to be related to the *Fangcheng* 方程 procedure used in a specific way by calendarists. The Fangcheng procedure is one of the nine branches of mathematics in the *Zhouli* 周禮 (*Rites of Zhou*), and

65 Zhu Yiwen,"Qin Jiushao dui *dayan*shu de suantu biaoda: jiyu *Shushu Jiuzhang* Zhao Qimei chaoben (1616) de fenxi"秦九韶對大衍術的筭圖表達：基於數書九章趙琦美鈔本(1616)的分析[Qin Jiushao's Writing of the 'Great Inference Procedure' Using Counting-Diagrams: Based on the Analysis of Zhao Qimei's Manuscript of the *Mathematical Book in Nine Chapter* (1616)], *Ziran kexueshi yanjiu* 自然科學史研究[*Studies in the History of Natural Sciences*], 36 (2),pp. 244–257; Zhu Yiwen, "How do We Understand Mathematical Practices in Non-mathematical Fields? Reflections Inspired by Cases from 12th and 13th Century China,"*Historia Mathematica*(June,2020).

66 Chemla, K. and Fox-Keller, E.,*Cultures without Culturalism: The Making of Scientific Knowledge*(Duke University Press, 2017).

its procedure was first recorded in the *Jiuzhang suanshu* 九章算術 (*Nine Chapters of Mathematical Procedures*). From a modern mathematical point of view, it is a method to solve linear equations. Like most ancient procedures, the procedure was carried out using counting rods.[67] It appears in most Chinese mathematical documents but did not receive any improvement until Qin Jiushao's work. Zhu Yiwen observed that the *Fangcheng* used in astronomy may be different from that presented in the *Nine Chapters*. The improvement by Qin Jiushao find its origin in astronomy. Qin Jiushao studied this procedure from calendarists. In order to further the reformation of the Kaixi calendar which happened during the Song dynasty, he decided to write down the procedure. Looking at calendars in official history, Zhu Yiwen noticed that numerous astronomical data were written, but only few procedures were recorded. Astronomical procedures were secret in order to keep the monopole over calendar production. Qin Jiushao established new connections among the *Fangcheng* procedures, the Procedure to Find One and the *Book of Changes*. The Procedure for All Numbers is inspired by the *Book of Changes*. Relating the procedures to one another gave him legitimacy to break the secrecy of the unwritten rules of astral sciences.

Qin Jiushao was within different cultures of computations that are respectively in mathematics and astronomy. For cultures of computations in mathematics, procedures were recorded in mathematical books with textual descriptions. As for the cultures of computations in astronomy, procedures were seldom recorded in calendars. Combining several elements from different cultures, Qin Jiushao created another kind of *dayan* procedure. He textualized the counting rods procedure, which will become known as the Chinese Remainder Theorem.

If we relate the Chinese remainder theorem to a culture of computation from calendar making in astronomy, then it is difficult to ignore Indian connections. Although it is quite impossible to imagine Qin Jiushao in direct contact with

67 Chemla, K. and Guo, S., *Les Neufs Chapitres: Le Classique des Mathematiques de la Chine ancienne et ses commentaires* (Dunod,2004), pp. 599-659.

Sanskrit culture, there are connections. Foreign contacts through the spread of Buddhism, which began during the last decades of the Han dynasty, continued –in art, sculpture, medicine, the sciences as well as religion. For instance, Fa Xian, the famous Buddhist pilgrim, set out for India in 399 and travelled the length and breadth of Northern India and over central Asia for fifteen years. Fa Xian lived during a period which saw two important mathematicians: Sun Zu (c. 300), in whose work we find the beginnings of indeterminate analysis, and Zu Chongzhi 祖冲之(c. 450), who accurately approximated Pi to be equal to 355/113.

There is only fragmentary evidence of Chinese-Indian cultural and scientific contacts before the rise of Buddhism around the 4th century AD. We do know that Xuan Zang 玄奘(c. 650 AD) and a number of Chinese Buddhist scholars, among whom early travellers such as Fa Xian, made their pilgrimage to holy places in India, bringing back many texts for translation. Among the places they visited were monasteries such as Nalanda and Taxil, which were Indian centres of scholarship not only in religion but in medicine, astronomy, and mathematics as well. Few of the writings or commentaries by these Buddhist pilgrims from China have been examined for what they may reveal about Indian sciences, the main interest being in their religious and sociological content. There was also evidence of Chinese diplomats posted to the court of the Guptas in India around the 5th century AD; and from the 7th century, there is evidence that translations were made of Indian astronomical and mathematical texts, such as *Po-luo-men Suanfa* 婆羅門算法 (*Brahman Arithmetical Rules*) and *Po-luo-men Suanjing* 婆羅門算經 (*Brahman Arithmetical Classic*) mentioned in records of the Sui dynasty (581–618). These works are no longer extant, and thus it is difficult to assess how influential they were on Chinese science. However, there is clearer evidence of Indian influence on Chinese astronomy and calendar making during the Tang dynasty. Indian astronomers were employed in the Imperial Bureau of Astronomy and charged with the tasks of preparing accurate calendars, some of which contain the names of Indian astronomers. One of the Indians, whose Chinese name was Xi Da (Siddhartha), was reputed to have constructed in 718 AD a calendar based on the

Indian *Siddhanta* of Varāhamihira (c. 550 AD), on the orders of the first emperor of the Tang dynasty. The text contains sections on Indian numerals, operations, and sine tables. There are also sine tables at intervals of 3deg.45' for a radius of 3438 units, which are the values given in the Indian astronomical texts *Āryabhaṭīya* and Sūrya-Siddhānta. This is the earliest record of a Chinese science table in any Chinese text. The works of Michio Yano[68] investigate several texts containing computations in astral sciences translated from Sanskrit into Chinese and preserved in Japanese monasteries since the 9th century. The first is the *Jiuzhili* 九執曆 (Navagraha Calendar) translated into Chinese in 718 by an Indian living in China whose excerpt are reminiscent of the *Pañcasiddhāntikā* by Varāhamihira (6th century). The second is *Qiyao rangzai jue* 七腰攘災決 (Secrets of Avoiding Disasters According to the Seven Planets). A Brahman originating from West India compiled it in 9th century. His name has been translated into Chinese and the text made its way to Japan in 865. The third is *Su yao jing* 宿曜经 (Canon of Lunar Lodges and Planets), which is also an Indian astrological text translated into Chinese from the mid-8th century. Yano gives an objective description of the content, text and mathematical formulae, and their relation with Sanskrit astronomical texts available in India. However, there is no philological interpretation. When one looks at the second wave of Buddhism, it becomes even clearer that conceptualizing cultures as nations is flawed. The spread of medieval Sanskrit mathematical astronomy into the northern Himalayas apparently occurred around the time of the so-called "second transmission" or revitalization of Buddhism in Tibet in the 11th century, largely influenced by Esoteric Buddhism in India. Its chief vehicle seems to have been the Sanskrit Buddhist work *Kala-cakra-tantra*, which was translated into Tibetan and Mongolian. The Indian mathematical astronomy therein described became the foundation of traditional calendars in Tibet, Mongolia, and Bhutan, which coexisted with a different form of

68 Yano Michio, "The Chiuchih-li and the Ārdharātrika-pakṣa,"*Journal of Indian and Buddhist Studies*, Vol.27, No.2, pp. 953-956; Yano Michio, "The Ch'iyao jang-tsai-chueh and its Ephemerides," *Centaurus* 29, pp.28-35;Yano Michio,"The Hsiu-yao Ching and its Sanskrit Sources," *History of Oriental Astronomy* (Cambridge University Press,1987), pp. 125-134.

mathematical astronomy derived from China.[69] We see here that it is difficult to speak of simple transmission and that it is difficult to apply the concept of borders.

Another example, Chemla and Keller,[70] also illuminates a number of concepts and operations concerning quadratic irrationals in India and China, which is opposed to the usage in Greek sources. Despite noted differences, they describe a cluster of similar features in the conception and use of quadratic irrationals in 7th century India and in first to 3rd century China. The same author[71] illustrates the complexity of transmission of knowledge between East and West. Her study of the Rule of False Double Position shows a direct transmission from China to Arabic-speaking worlds. She observed that stability of the way of expressing these rules and of applying them makes it difficult to believe in independent discoveries. Everything indicates a continuation from China, to Arabic-speaking world then into Europe, however the sophistication of the older Chinese sources seemed not to have been retained in the process of transmission. Moreover, after observing some Arabic and Chinese mathematical texts, Chemla[72] showed that in Arabic sources of the twelfth century, algorithms are similar to procedures contained in Chinese and, independently, in Indian sources. If one pays attention to the way in which algorithms for root extraction are set up and results are generated, there seem to be two distinct traditions of Arabic mathematics. One of these traditions, embodied by al-Uqlidisi and by al-Khwarizmi, shares common features with all

69 Ohashi Yukio, "Remarks on the origin of Indo-Tibetan astronomy," in Selin, Helaine (ed.), *Astronomy Across Cultures: The History of Non-Western Astronomy* (Dordrecht: Kluwer, 2000), pp. 341-369.

70 Chemla, K. and Keller, A.,"The Sanskrit karaṇīs and the Chinese *mian (side).* Computation with Quadratic Irrationals in Ancient China and India," in Y. Dold-Samplonius, J.W. Dauben, M. Flokerts, B. Van Dalen (Eds), *From China to Paris : 2000 Years Transmission of Mathematical Ideas* (Boethius:Franz Steiner Verlag,2002), pp. 87-132.

71 Chemla, K.,"Reflections on the world-wide history of the rule of false double position, or how a loop was close," *Centaurus*, Vol.39(1997), pp. 97-120.

72 Chemla, K.,"Nombre, opérations et equations en divers fonctionnements: quelques méthods de comparaison entre des procedures élaborées dans trois mondes diffrérents," In I. Ang and P.E Will (eds.), *Nombres, astres, plantes et viscères. Sept essais sur l'histoire des sciences et des techniques en Asie orientale*(Paris : College de France, Institut des Hautes Etudes Chinoises,1994) (Memoires de l'Institut des Hautes Etudes Chinoises, XXXV), pp. 1-36.

Indian algorithms and none of the Chinese ones; the other tradition, embodied by Kushyar ibn-Labban and his student Nasawi, shares the opposite features with all the Chinese texts and none of the Indian ones. Following this basis, Chemla suggested abandoning the hypothesis that the corpus of Arabic texts is organized into a linear continuation from Greece and India to Arabic worlds. She distinguished two traditions among Arabic arithmetic, indeed one linked to India, but the other to China. Despite there being no historical evidence of any direct connection, there is a set of clues on possible mathematical connections between China and the Arabic world around the eleventh and the twelfth centuries about equations

That is, this raises the question the circulation of mathematical concepts and practices between China and India and argue that a whole part of the international history of mathematics remains unexplored. In 2016, Hoyrup showed that milieus matter more than nationality.[73] He showed that some problems have circulated as ‘subscientific’ mathematics, more precisely as professional riddles belonging to an environment of mathematical practitioners. Travelling merchants and accountants do not share the same culture. Arithmetical riddles might be carried along the Silk Road within a transnational network of travelling merchants and were exchanged as campfire for fun or challenge. Some geometrical knowledge travelled too, but to a lesser extent and in a different context. In this context, it would be important to understand how epistemological cultures were in contact in China and India. This should include an investigation of primary sources, an investigation of mathematical practices and cultures of computation, and an investigation of the construction of comparisons. These new findings on Qin Jiushao and on Tibetan astronomy have provided new interpretations on what is called “influences”. They

73 Hoyrup, J. ,“Seleucid, Demotic and Mediterranean Mathematics versus Chapter VIII and IX of the Nine Chapters: Accidental or Significant Similarities?,”《自然科學史研究》*Researches in the History of Natural Sciences*, Vol.35 (2016), pp. 463-476.

invite us to re-investigate the history of exchange between Chinese and Indian areas.

There are no direct connections with one mathematician copying and translating another mathematician's materials, but instead a steady stream of exchange between north (Mongolia) and south (Tibet, India), East (China) and West (India). There are practices borrowed and mixed, combined with other practices as we see in Qin Jiushao's case. Cultures of computation change along with practices and concepts. Practices and concepts also change along with the practitioner's work. An unawareness of these streams of exchange has resulted in a situation whereby the relations between practices in mathematics and astronomy are insufficiently treated in the field of history of mathematics. To make matters worse, the separation of the two fields (mathematics and astronomy) led to a feeling of contempt towards the algorithmic shape of Chinese mathematics in Western historiography. However, if one sees the relation through the prism of epistemological cultures, one needs to reopen the debate. Cultures in this case do not equate to nation or language, but rather to specific practices belong to specific milieu. That is to say that there are as many differences between "China" and "India" as there are between an astronomer and an accountant. Sources are written in relation to specific cultures of mathematical practices.

III. Heuristic comparison: From the point of view of cognitive history

There is another way to use comparison that shows promise when we investigate differences. This section deals with identifying different kinds of cognition. We have long known how important the contribution of history of mathematics has been for mathematics education – and then link it to psychology... Since Piaget, we have known that children's logic is different from that of adults, and we know that the teaching of mathematics should be adapted to the cognition of children. The main question in mathematics education is to decide *how* it should be adapted. History of mathematics can show the difficulties encountered by our ancestors facing new mathematical objects, how they reacted to new concepts and how they were creative in their own answers. Using philology to read ancient texts also provides some clues on how texts were composed and read and on the link between texts and mathematical practices. This type of academic practice reveals some cognitive aspects different from or similar to ours. Naturally, we cannot deduce that the cognition of ancient mathematicians is the same as that of children studying new mathematical objects. Yet, it seems it is possible to measure the possible gaps between our thinking and other types of thinking. To reconstruct the logic of others, we must be able to think of unexpected relations. Comparison is a way to think the unexpected. The comparative method applied to "non-Western" mathematics is a way to open a window on something different to the usual axiomatic-deductive style of thinking.

Stengers already discussed the epistemological problems of cognitive history of science in "Quelle histoire des sciences?" (What kind of history of sciences?) in 1984.[74] But recently Netz has offered a new insight on practices which may have an influence on the cognitive possibility of science.[75] He opened a door for the interpretation of results generated by philological methods applied to scientific discourses and related diagrams, showing that a Cognitive History of Science is possible. It would be a philosophical anthropology of scientific reason that holds that styles of scientific thinking are grounded in innate potentials, many of which are cognitive, which have to be discovered in the course of human history. This is something we already find in Piaget to some extent. Piaget did not start his work in children psychology per se. His aim was to explore the mechanisms responsible for cognitive development and children were living laboratories.

74 Stenger, I., "Quelle histoire des sciences ?,"*Histoire des Sciences et Psychogenese*. (Geneva:Fondation Archives Jean Piaget,1983).

75 Netz, R., *The Shaping of Deduction in Greek Mathematics. A Study in Cognitive History* (Cambridge University Press,1999).

Netz's research is to be related to the concept of styles found in historical epistemology, according to Hacking.[76] Styles are constituted by methods and objects (not by disciplines). They introduce new objects and new criteria that determine whether those objects are perceived as true or false. They define the criteria for truth-telling in their domain. They are autonomous, "self-authenticating" (they introduced their own criteria of evidence), proof and demonstration and they are punctuated by moments of crystallization. This philosophy of style implies cognitive foundations and cultural history: styles are grounded in human cognitive and physiological capacities, which are universal. Meanwhile, scientific styles are also the product of cultural innovation and evolution. In case of mathematics, there is a module (or group of modules) dedicated (i) to spatial configurations, (ii) to numerical or arithmetical reasoning, (iii) to algorithmic and combinatorial reasoning. Hacking does not favor the idea of continuity in history of sciences. On the contrary, he argues that there are sharp moments of crystallization, moments in the evolution of style of scientific thinking that are irreversible in effect. Such a moment is also accompanied by a 'legend', for example Galileo for hypothetical modelling or Euclid for axiomatic deductive style. Spatial geometrical thinking involves cognitive capacities different from arithmetical, combinatorial and algorithmic reasoning. The evolution of this style in the 9th century on the Arabian Peninsula is represented by Al-Khwarizmi as a moment of crystallization for combinatorial thinking. According to Hacking, mathematical objects are born out of perspicuous proof, not calculation. The problem is that by opposing calculation and proof, he is reducing the scope of mathematical style of thinking to being exclusively 'axiomatic-deductive' and to a certain type of proof, thus excluding proof of correction of algorithms. His way of describing algorithms echoes Libbrecht's described above: deduction on one side, algorithm on the other. Here, philosophy echoes culturalization in history of sciences.

76 Hacking, I., *Scientific Reason* (Taipei,Taiwan: NTU Press,2009).

Chemla[77] has already shown that computational problems whose sequence of procedure aims at the proof of correction of an algorithm are experiences of generality. In a recent study Pollet[78] shows that geometry in *Yigu yanduan* 益古演段 (the Development of Pieces of Areas According the Collection Augmenting the Ancient Knowledge) written by Li Ye 李冶 in 1259, is all about seeing and grasping mathematical arguments on the construction of polynomials and equations. The interesting point is that all demonstrations rely on the capacity of readers to visualize transformations of geometrical figures. All proofs rely on the ability of the practitioner to see movements and interactions of geometrical areas.

The problems in the *Yigu yanduan* are related to computations of circular and square areas. However, problems that looks like practical surveying are the frame of expression of an abstract procedure. In the first problem - computing the diameter of a circular pond knowing the side and area of an outer field-, Li Ye adds geometrical figures and the *tian yuan* algorithm. The operation on geometrical figures consists of cutting, pasting areas and reading them as piled layers. In other words: by drawing and visualizing the transformations of areas, the mathematician justifies the construction of the terms of the equation. The diagram is therefore the main tool for the construction of an equation. By legitimizing the origins of the areas, it consequently confirms the validity of the procedure. The diagram is at once an interpretation, a rewording of the data laid out in the statement of the problem and a way to visualize the equation. It provides verification of how the data of the statement are transformed into an equation, so value of the diagram is also demonstrative. After 64 problems, the practitioner sees the analogical connection among problems. The combination of multiple figures guides the practitioner to understand the generality of the procedure. Generality thus

77 Chemla, K.,"What is the content of this book? A Plea for Developing History of Sciences and History of Text Conjointly," Chemla Karine (ed), *History of Science, History of Text*, (Dordrecht-Boston-London: Kluwer Academic Publishers,2004) pp. 201- 230.

78 Pollet, C. , *The Empty and the Full: Li Ye and the way of mathematics* (Singapore:World Scientific,2020).

expresses itself through the organization of the order of problems, which appears after a mental visualization of the transformation of diagrams.

The treatise, which looks like a repetitive list of algorithms, is in fact a dynamic structure guiding the practitioner to an experience on generality. Interestingly, we see a combinatorial system hidden behind the list of the algebraic procedure and their geometrical solutions. This combinatorial thinking related to algebra is at least as old as the 9th century in China. This style must be rooted in the Taoist network and in the philosophy of Changes, which makes its origin even older. This combinatorial thinking reminds one of practices at work in magic squares. It is accepted that apart from Zhang Zhao (1650 AD) who showed the first complete magic square of order 10 and Bao Jishou (1880 AD) who constructed three-dimensional magic cubes, spheres and tetrahedrons, there was hardly any innovation since Yang Hui 楊輝 (13th century). I showed how Li Ye and Yang Hui's algebra is related to practice of combinatorics and how their lists of geometric problems is more related to the investigation of magic square than to teaching algebra.[79] That is algebra, combinatorics and geometry were already connected. We can also see this phenomenon in India, but there are meaningful differences.

First of all, many Sanskrit texts are shaped like an enumeration. The texts are usually organized around the classification of mathematical objects, procedures (arithmetic or algebraic) and resolution of equation. It is an enumeration of topics and their combination of objects. As if enumerating all the parts composing what we name "algebra", it constructs an analytical inventory of the fundamental realities constituting the field of mathematics. For instance, the *Bījagaṇitāvataṃsa* written by Nārāyaṇa in 14th century is shaped like a list of topics followed by many of examples. Nārāyaṇa is the author of two works: interestingly one contains an important chapter on algebra and the other on magic squares.

79 Pollet, C. , *The Empty and the Full: Li Ye and the way of mathematics* (Singapore:World Scientific,2020).

Nārāyaṇa is the main source quoted when it comes to magic squares, permutation and combination in India. The other book, *Gaṇitakaumudī* (GK. "moonlight of mathematics" 1356) is always quoted as a masterpiece on combinatorial thinking in India. GK Ch. 13 contains rules on combinatorics (permutations, combinations, partitions, binomial and multibinomial coefficients, sequence of polynomial coefficients etc.), series and related topics. GK Ch. 14 focuses on magic squares instead. Magic squares have not been dealt with by any other earlier Hindu mathematicians.[80] Magic squares have been known in India since at least the 6th century in Varāhamihira. Nārāyaṇa is the first to introduce magic squares as a topic in *pāṭī-gaṇita*, 'the mathematics of algorithm' (R.XIII. 1-9): "For the pleasure of mathematicians, [I] now describe briefly the *anka-pāśa* (concatenation of numbers, i.e., combinatorics) where bad, wicked and intoxicated mathematician's vanity shatters. The knowledge of *anka-pāśa* is very useful in dramatics, prosody, medicine, garland making, architecture and mathematics". Nārāyaṇa mentions dramatics and prosody first. Indeed, there is an explicit tradition of combinatorial thinking is Sanskrit metric.

The basic units in Sanskrit prosody are syllables with one *mātrā* (syllabic instant), called *laghu* (light), and syllables with two *mātrā*, called *guru* (heavy). Pingala (+/- 200 BC) summarizes these rules in *Chandahsūtra* (Prosody rules). Nārāyana extends and generalizes these *pratayas* of Sanskrit prosody to combinatorial. From a combinatorial point of view, the idea is to determine the configurations of forms under given constraints. All rules stated in versified *sūtra* are a progression towards generalization. Interestingly, Nārāyaṇa gives exhaustive rules for constructing not only ordinary magic squares but also variants made by combining multiple magic squares. Here, combinatorial thinking has other roots than its Chinese counterpart. Yet, in both cases they will be connected to numbers and algebra in the same time period. In Nārāyaṇa's works, one finds more or less the same ingredients of numerology and combinatorial analysis as in the Chinese

80 Kusuba, T.,"Combinatoric and Magic Square in India: A Study of Nārāyana Pandita's Ganitakaumudi", Chapters 13-14(Ann Arbor,1994).

sources.[81] It is interesting to note that the works of Thabit ibn Qurra (850 AD), al-Gazzali (1075 AD), and al- Biruni (1225 AD) contain the same perspective. Independently from Chinese authors, Nārāyaṇa also produced work on both magic squares and algebra.

What we have then are mathematicians in two distinct cultural spheres that show a relation between algebraic and combinatorial thinking. But, in Chinese, we have mathematical objects which are not described, but rather defined by their function in movement in geometrical figures. In Sanskrit, some enumerations express properties that are assigned to procedure. We can link these observations to Inge Schwank's research on algorithmic thinking cognitive structures.[82] Schwank shows that it is possible to identify two types of cognition in algorithmic thinking: A predicative one focusing on structures and their description, whose expression is static, and a functional one interested in processes and effects, whose expression is dynamic. In both cases, we see algorithm and lists, but their way of processing unveil different cognitive practices.

Comparing Sanskrit and Chinese sources is a key to understanding what is called algebra in India and its relations to China through the prisms of practices. These two areas' study is promising for history of cognitions: from spatial configuration to combinatorial and algebraic thinking. We can agree on an evolution of cognition as proposed by Hacking, but style should not be reduced to axiomatic deductive style. The distinction between functional and predicative cognition seems promising. The division between cultures, calculation and proof must be re-investigated.

81 Joseph,G.G.,*The Crest of the Peacock. Non-European Roots of Mathematics*, p.214.

82 Schwank, I., "On the Analysis of Cognitive Structures in Algorithmic Thinking". *Journal of Mathematical Behavior*. Vol.12(1993).

IV. Conclusion

A comparison between Chinese and Indian algorithmic constructions of polynomial equations shows the implication of deductive skills. The use of analogy in Chinese texts or algorithms in Indian texts is argumentative since these are verification and demonstration practices. The articulation of a list of operations conveys mathematical meaning. Li Ye relied on a dynamic reading of diagrams to focus on equations and this should be related to functional cognition. Nārāyaṇa preferred descriptions of structures and relations among objects (unknowns/indeterminates). Here, the function is predicative. Li Ye used visualization in geometry and combinatorics to reach generality. This practice is related to the Taoist investigations of Changes. Nārāyaṇa also used combinatorics from the context of prosody. This result has implications on what we define as 'mathematical culture'. If culture consists of patterns - explicit and implicit – and of behavior that has acquired and transmitted symbols; if culture constitutes the distinctive achievements of human groups, including their embodiments in artifacts then the essential core of culture consists of traditional (historically derived and selected) ideas and especially their attached values.[83] If we talk about behavior related to the interpretation of symbols, then identifying practices and cognitive structures in ancient text is extremely significant. The description of culture as epistemic culture yields an important tool for carrying out conceptual history. This works in several ways: cultures change in relation to the conceptual work done; concepts change in relation to how actors work. Therefore, there cannot be any influence of "China" or "India". Instead, there are influences from calendarists to mathematicians. The question of influence or comparison presented in coincidence with nations is nothing but culturalism. Culturalism is a perspective that views scientific cultures and practices in terms of 'essential' cultures of the country in question and that seeks to understand their development through

83 Kroeber and Kluckhohn, *Culture: A critical review of concept and definitions* (Papers of the Peabody Museum of American Archeology and Ethnology, Harvard University,Vol. XLVII. N.1) (USA:Cambridge, Massachusets,1952).

macroscopic consideration of such culture.[84] It assumes that the 'influenced' culture is always passive and depicts essential culture as unique to the area and fundamentally different from cultures from other regions. It supposes a form of cultural determinism about science as well. That is the assumption that cultural differences can explain the existence of different scientific practices in other places. Instead, one needs to approach cultures by recognizing their heterogeneity and transregionality.

84 Ito, K. ,"Cultural Difference and Sameness: Historiographic Reflections on Histories of Physics in Modern Japan," in Chemla, K. Fox-Keller, E.(eds.), *Cultures without Culturalism: The Making of Scientific Knowledge* (Duke University Press, 2017), p.49.

Bibliography

BIERNATZKI, K.L.

1856 "Die Arithmetik der Chinesen,"*Journal für reine und angewandte Mathematik*, 52 (1856) , pp. 59-94.

BIOT, E.

1839 "Table générale d'un ouvrage chinois intitule Souan-Fa Tong-Tsong, ou traité complet de l'art de compter," *Journal asiatique*, 3rd series, No.7, (1839), pp. 193-217.

BECKER and HOFMANN.

1951 *Geschichte der Mathematik*,1951.

CAJORI, F.

1919 *A History of Mathematics*, 2nd edition (New York,1919).

CANTOR, M.

1908 *Vorlesungen über die Geschichte der Mathematik*, 4vols/ Leipzig, 1880-1908).

CHEMLA, K.

1994 "Nombre, opérations et equations en divers fonctionnements: quelques méthods de comparaison entre des procedures élaborées dans trois mondes diffrérents," In I. Ang and P.E Will (eds.), *Nombres, astres, plantes et viscères. Sept essais sur l'histoire des sciences et des techniques en Asie orientale*(Paris : College de France, Institut des Hautes Etudes Chinoises,1994) (Memoires de l'Institut des Hautes Etudes Chinoises, XXXV), pp. 1-36.

1997 "Reflections on the world-wide history of the rule of false double position, or how a loop was close," *Centaurus*, Vol.39(1997), pp. 97-120.

2004 "What is the content of this book? A Plea for Developing History of Sciences and History of Text Conjointly," Chemla Karine (ed), *History of Science, History of Text*, (Dordrecht-Boston-London: Kluwer Academic Publishers,2004) pp. 201- 230.

CHEMLA, K. and KELLER, A.

2002 "The Sanskrit karaṇīs and the Chinese *mian (side).* Computation with Quadratic Irrationals in Ancient China and India,*"* in Y. Dold-Samplonius, J.W. Dauben, M. Flokerts, B. Van Dalen (Eds), *From China to Paris : 2000 Years Transmission of Mathematical Ideas* (Boethius:Franz Steiner Verlag,2002), pp. 87-132.

CHEMLA, K. and GUO, S.

2004 *Les Neufs Chapitres: Le Classique des Mathematiques de la Chine ancienne et ses commentaires* (Dunod,2004).

CHEMLA, K. and FOX-KELLER, E.

2017 *Cultures without Culturalism: The Making of Scientific Knowledge* (Duke University Press, 2017).

CLARK, W. E.

1930 *The Aryabhatiya of Aryabhata, translated into English with notes* (Chicago: University of Chicago Press,1930).

DATTA, B.

1932 *The Sciences of the Sulbas: A study in early Hindu geometry* (Calcutta: Calcutta University Press,1932).

DATTA, B. and SINGH, A. N.

1935 *History of Hindu Mathematics, A Source Book*, Part I and II, (Bombay, India: Asia Publishing House,1935).

DAUBEN, J. W. (ed.)

2000 *The History of Mathematics from Antiquity to the Present: A Selective Annotated Bibliography (Revised Edition)*, (New York: Garland,198), (Revised CD_ROM edition published by American Mathematical Society,2000).

GANGULI, S.

1929 "Notes on Indian Mathematics: A criticism of G. R. Kaye's interpretation," Isis 12(1929),pp.132-145.

1931 "India's contribution to the Theory of Indeterminate Equations of the Frist Degree," *The Journal of Indian Mathematical Society* (Madras),19(1931), pp.110-169.

HACKING, I.

2009 *Scientific Reason* (Taipei,Taiwan: NTU Press,2009).

HANKEL, H.

1874 *Geschichte der Mathematik im Altertum und Mittelalter*(Leipzig,1874).

HAYASHI, T.

1905 "Brief History of Japanese Mathematics," *Nieuw Archief voor Wiskunde*. 6(1905), pp. 296-361.

HOYRUP, J.

2016 "Seleucid, Demotic and Mediterranean Mathematics versus Chapter VIII and IX of the Nine Chapters: Accidental or Significant Similarities?,"《自然科學史研究》*Researches in the History of Natural Sciences*, Vol.35 (2016),pp. 463-476.

ITO, K.

2017 "Cultural Difference and Sameness: Historiographic Reflections on Histories of Physics in Modern Japan," in Chemla, K. Fox-Keller, E.(eds.), *Cultures without Culturalism: The Making of Scientific Knowledge* (Duke University Press, 2017).

JOSEPH, G. G

2011 *The Crest of the Peacock. Non-European Roots of Mathematics*, 3rd edition (Princeton and Oxford:Princeton University Press, 2011).

KAYE, G. R.

1908 "Note on the Indian Mathematics. No. 2. Aryabhaṭa," *Journal of the Asiatic Society of Bengal* IV 8(1908), pp. 111-114.

1915 *Indian Mathematics*(Calcutta, 1915).

KERN, H.

1874 *The Aryabhaṭīya with the commentary of Bhaṭadīpika of Parameśvara* (Leiden,1874).

KUSUBA, T.

1994 "Combinatoric and Magic Square in India: A Study of Nārāyana Pandita's Ganitakaumudi", Chapters 13-14(Ann Arbor,1994).

KROEBER and KLUCKHOHN

1952 *Culture: A critical review of concept and definitions* (Papers of the Peabody Museum of American Archeology and Ethnology, Harvard University,Vol. XLVII. N.1) (USA:Cambridge, Massachusets,1952).

LAM, Lay Yong and ANG, Tian Se

2004 *Fleeting Footsteps: Tracing the Conception of Arithmetic and Algebra in Ancient China*,Revised edition (Singapore :World Scientific,2004).

LEONARDO FIBONACCI

2002 *Liber Abaci* (1202), Laurence E.Sigler (trad.), *Fibonacci's Liber Abaci : A Translation Into Modern English of Leonardo Pisano's Book of Calculation* (Springer-Verlag, 2002).

LIBBRECHT, U.

1973 *Chinese Mathematics in the Thirteenth Century: The Shu-shu chiu-chang of Ch'in Chiu-shao* (Cambridge, Massachusetts :The MIT Press,1973).

LI, Yan

1958 *Zhongguo kexue dagang* 中國數學大綱 [Outlines of Chinese Mathematics], 2 vols (Beijing: Kexue chubanshe,1958).

LI, Yan and DU, Shiran

1987 *Chinese Mathematics: A Concise History*, Translated by Crossley J.N and Lun AA. W-C., (Oxford: Clarendon Press,1987).

LIM, T. S. L. and WAGNER D. B.

2017 "The Continuation of Ancient Mathematics: Wang Xiaotong's Jigu suanjing,"*Algebra and Geometry in the 7th-Century China* (Copenhagen: NIAS Press, 2017).

LORIA, G.

1922 “Documenti relativi all’antica matematica dei cinesi,”*Archeion*, 3(1922), pp.141-149.

1929 *Storia delle matematiche dall’ alba della civiltà al secolo XIX*, 3 vols (Turin, 1929).

LU, Peng 呂鵬 and JI, Zhigang 紀志剛

2019 “Yindu kutaka xiangjie ji qi yu da yan zongshu shu bijiao xin tan” 印度庫塔卡詳解及其與大衍總數術比較新探 [Detailed Explanation of Kutaka, India and a New Probe into Comparison with Dayan Summarization],《自然科學史研究》*Researches in the History of Natural Sciences*, Vol.38, Issuse 2(2019), pp. 172-188.

MATTHIESSEN, L.

1876 “Vergleichung der indischen Cuttaca und der chinesischen Tayen-Regel,” in *Sitzungsberichte der mathematisch-naturwissenschaftlichen Section in der 30. Versammlung deutscher Philologen und Schulmänner in Rostock*, *1975, 75* (Leipzig, 1876).

MAJUMDAR, N. K.

1912 “Aryabhata’s rule in relation to indeterminate equations of the first degree,” *Bulletin of the Calcutta Mathematical Society*,3(1911-1912).

1914 “On Chinese Indeterminate Analysis,” *Bulletin of the Calcutta Mathematical Society*, 5(1913-1914) .

MARTZLOFF, J. C.

1988 *Histoire des Mathématiques Chinoises* (Paris : Masson,1988).

MIKAMI, Y.

1913 *The Development of Mathematics in China and Japan* (Abhandlungen zur Geschichte der mathematischen Wissenschaften, n.30), Leipzig, 1921 (reviewed by H. Bosmans, 1913).

NEEDHAM, J.

1954 *Science and Civilisation in China*, 7 vols(Cambridge,1954).

NETZ, R.

1999 *The Shaping of Deduction in Greek Mathematics. A Study in Cognitive History* (Cambridge University Press,1999). Ideas in Context 51.

OHASHI, Yukio.

2000 "Remarks on the origin of Indo-Tibetan astronomy," in Selin, Helaine (ed.), *Astronomy Across Cultures: The History of Non-Western Astronomy* (Dordrecht: Kluwer, 2000), pp. 341-369.

POLLET, C.

2018a "Reading Algorithms in Sanskrit: How to Relate Rule of Three, Choice of Unknown and Linear Equation?," in A. Volkov and V. Freiman (eds.), *Computations and Computing Devices in Mathematics Education before the Advent of Electronic Calculators* (Mathematics Education in the Digital Era Series, Springer Publishers,2018).

2018b "Interpreting Algorithms written in Chinese from the Point of View of Comparative History," in A. Volkov and V. Freiman (eds.), *Computations and Computing Devices in Mathematics Education before the Advent of Electronic Calculators* (Mathematics Education in the Digital Era Series, Springer Publishers,2018).

2020 *The Empty and the Full: Li Ye and the way of mathematics* (Singapore:World Scientific,2020).

POLLET, C. and YING, J-M.

2017 "One quadratic equation, different understandings: the 13th century interpretation by Li Ye and later commentators in the 18th and 19th centuries" *Journal for History of Mathematics*,Vol. 30 No.3 (June 2017), pp. 137-162.

QIAN, Baocong.

1966 *Song yuan shuxue lunwenji* 宋元數學史論文集[Collection of essays on the history of Song and Yuan mathematics] (Beijing,1966).

RODET, L.

1879 *Leçons de calcul de Aryabhata*(Paris: Imprimerie nationale,1879).

Siu Man-Keug & Chan Yip-cheung

2012 "On Alexander Wylie's Jotting on the science of the Chinese Arithmetic," *Hisrory and Pedagogy of Mathematics 2012*, (Daejeon, Korea: DCC, July 16-20 2012).

SMITH, D.E.

1931 "Unsettled Questions concerning the Mathematics of China," *The Scientific Monthly*., Vol. 33, No. 3 (Sep., 1931), pp. 244-250.

SARTON, G.

1947 *Introduction to the History of Sciences*, 3 vols, No. 376. (Baltimore:Carnegie Institution Publ,1927-1947).

SEN, S. N.

1962 "Study of Indeterminate Analysis in Ancient India". *Proc. 10th International Congress of the History of Sciences* (Ithaca, 1962).

SENGUPTA, P. C.

1927 "The Aryabhatiyan, translation, "*Journal of the Departments of Letters of the University of Calcutta*, XVI (1927).

STENGER, I.

1983 "Quelle histoire des sciences ?,"*Histoire des Sciences et Psychogenese*. (Geneva:Fondation Archives Jean Piaget,1983).

SCHWANK, I.

1993 "On the Analysis of Cognitive Structures in Algorithmic Thinking". *Journal of Mathematical Behavior*. Vol.12(1993).

Terquem, O.

1863 *Nouvelles Annales de Mathematiques* (2e ser., 1863).

TROPFKE, J.

1924 *Geschichte der Elementarmathematik*,7 vols (Leipzig,1921-1924).

VAN HEE.

1912 "L'algèbre chinoise," *Toug Pao*, 13(1912).

1932 "Le classique de l'ile maritime, ouvrage chinois du IIIe siècle," *Quellen und Studien zur Geschichte der Mathematik* (Part B: Astronomie und Physik), 2(1932).

WYLIE, A

1852 "Jotting on the Science of the Chinese: Arithmetic," *North China Herald*, (Aug.-Nov, 1852). Nos. 108, 111, 112, 113, 116,117, 119, 120, 121. Repr. *Copernicus*, 1882, 2, 169, 183. (For other reprints, see *Needham's Science and Civilization in China*, vol. 3, p.800).

YANO, Michio.

1979 "The Chiuchih-li and the Ārdharātrika-pakṣa,"*Journal of Indian and Buddhist Studies*, Vol.27, No.2, pp. 953-956.

1986 "The Ch'iyao jang-tsai-chueh and its Ephemerides," *Centaurus* 29, pp.28-35.

1987 "The Hsiu-yao Ching and its Sanskrit Sources," *History of Oriental Astronomy* (Cambridge University Press,1987), pp. 125-134.

YUSHKEVICH, A. P.

1955 "O dostizenijax kitajskix ucenyx v oblasti mathematiki," (On the achievements of Chinese scholars in the field of mathematics), in *Iz Istorii Nauki I Texniki Kitaja* (Essays in the History of Science and Technology in China), Moscow.

1964 *Geschichte der Mathematik im Mittelalter* (tr. from Russian)(Leipzig, 1964). English translation, *History of Mathematics in the Middle Ages*,Cambridge, Mass., and London, 1973.

ZHU, Yiwen.

2017 "Qin Jiushao dui *dayan*shu de suantu biaoda: jiyu *Shushu Jiuzhang* Zhao Qimei chaoben (1616) de fenxi"秦九韶對大衍術的筭圖表達：基於數書九章趙琦美鈔本(1616)的分析[Qin Jiushao's Writing of the 'Great Inference Procedure' Using Counting-Diagrams: Based on the Analysis of Zhao Qimei's Manuscript of the *Mathematical Book in Nine Chapter* (1616)], *Ziran kexueshi yanjiu*自然科學史研究[*Studies in the History of Natural Sciences*], 36 (2),pp. 244–257.

2020 “How do We Understand Mathematical Practices in Non-mathematical Fields? Reflections Inspired by Cases from 12th and 13th Century China,”*Historia Mathematica*(June,2020). https://doi.org/10.1016/j.hm.2020.04.004